

3rd International Meeting on Geometric Group Theory and Low Dimensional Topology

November, 3 and 4, 2022

Talks and Abstracts

Macarena Arenas

A cubical Rips construction

The Rips exact sequence is a useful tool for producing examples of groups satisfying combinations of properties that are not obviously compatible. It works by taking as an input an arbitrary finitely presented group Q , and producing as an output a hyperbolic group G that maps onto Q with finitely generated kernel. The “output group” G is crafted by adding generators and relations to a presentation of Q , in such a way that these relations create enough “noise” in the presentation to ensure hyperbolicity. One can then lift pathological properties of Q to (some subgroup of) G . Among other things, Rips used his construction to produce the first examples of incoherent hyperbolic groups, and of hyperbolic groups with unsolvable generalized word problem. In this talk, I will explain Rips’ result, describe a variation of it that produces cubulated hyperbolic groups of any desired cohomological dimension, and survey some tools and concepts related to these constructions, including classical and cubical small cancellation theories and cubulated groups.

Jonathan Barmak

On the metabelianization of Burnside groups

The Burnside group $B(d, n)$ is presented by d generators and the relators are all the n th powers. It is known that some of these groups are finite and some others are not, but for given d, n , the question in general is open. The existence of a largest finite quotient $R(d, n)$ of $B(d, n)$ (the Restricted Burnside Problem) was settled in 1991 by Zelmanov, but again little is known about these groups. The free metabelian group d generators is the quotient of $B(d, n)$ by its second derived subgroup. This is an approximation to $R(d, n)$. It is always finite and easier to study. However, the order of these groups is not known in general.

In this talk I will present invariants used to identify nontrivial elements of $M(d, n)$, and in particular we will improve known bounds for the order of these groups. Our invariants are based on the Winding invariant and have a strong combinatorial flavor.

Agustin Barreto

On the asphericity of some classes of LOTs of diameter 4

A LOT is an oriented tree with a labeling function from the edge set into the vertex set. One can associate to each LOT a group presentation and so the corresponding standard 2-complex. LOT complexes are potential counterexamples of Whitehead's Asphericity Conjecture. In the eighties Howie proved that all LOTs of diameter at most 3 are aspherical and extended his results to some LOTs of diameter 4. In this talk I will discuss a combinatorial method to study the asphericity of some classes of LOTs of diameter 4, focusing on those with exactly 3 non-extremal points, extending Howie's results. This is joint work with Gabriel Minian.

Martin Blufstein

Parabolic subgroups of Artin groups

Parabolic subgroups play a central role in the study of Artin groups. In this talk we will address two questions regarding them. First we will see that the intersection of parabolic subgroups is a parabolic subgroup when the Artin group is (2,2)-free and two-dimensional. Secondly we will show that, for all Artin groups, whenever a parabolic subgroup P is a subgroup of another parabolic subgroup Q , then P is a parabolic subgroup of Q . The second part of the talk is joint work with Luis Paris.

Ximena Fernandez

Morse theory for group presentations

Discrete Morse theory is a combinatorial tool to simplify the structure of a given (regular) CW-complex up to homotopy equivalence, in terms of the critical cells of discrete Morse functions. In this talk, I will present a refinement of this theory that guarantees not only a homotopy equivalence with the Morse CW-complex, but also a Whitehead's simple homotopy equivalence. Moreover, it provides an explicit description of the attaching maps of the critical cells in the simplified complex and bounds on the dimension of the complexes involved in the deformation. This result provides the suitable theoretical framework for the study of different problems in combinatorial group theory and topological data analysis. I will show an application of this technique that allows to prove that some potential counterexamples to the Andrews-Curtis conjecture do satisfy the conjecture. Moreover, the method can also be extended to filtrations of CW-complexes, providing an efficient algorithm for the computation of the persistent fundamental group of point clouds in terms of group presentations.

Fernandez, X. Morse theory for group presentations. arXiv:1912.00115

Wajid Mannan

An exotic presentation of Q_{28}

For a finite group G to have non-free stably free modules over its integral group ring, G must have a binary polyhedral quotient. Of the binary polyhedral groups themselves all but 7 have such stably free modules. In the early 2000's topologists such as Rudolph Beyl, Nancy Waller and Francis Johnson were exploiting these stably free modules to construct cohomologically 2-dimensional 3-complexes which are not homotopy equivalent to the standard presentations of their fundamental groups. This begged the question: Are they homotopy equivalent to finite group presentations at all? There did not seem to be any reason to think they would be, given their number theoretic origins. However the existence of such a complex which is not homotopy equivalent to a finite group presentation is a major conjecture, known as Wall's D(2) problem.

For the simplest of these complexes, known as Nancy's Toy, I will discuss how we found a homotopy equivalent finite group presentation.

Gabriel Minian

I-test and LOTs

Some years ago, in collaboration with J. Barmak, we introduced the I-test, which is a method for analyzing asphericity of group presentations. In contrast to the different variations of the weight test, this method is not based on curvature. In this talk I will present new applications of the I-test to the study of presentations arising from LOTs (labeled oriented trees), which are considered test cases for Whitehead asphericity conjecture.

John Nicholson

Wall's problems on 2-complexes, some progress and new directions

In 1977, C. T. C. Wall compiled a list of nine problems concerning the topology of 2-complexes. It is widely believed that there exist counterexamples to all of them, but no such examples have been found since the first one was found back in 1990. The aim of this talk will be to present a counterexample to a second problem on Wall's list and to discuss various analogues of these problems in 4-manifold theory which are yet to be fully explored.

Gerald Williams

Generalized polygons and star graphs of cyclic presentations of groups

The concept of "special" presentations was introduced by Howie as a particular class of $C(3)-T(6)$ (small cancellation) presentations, specifically those whose relators have length $k = 3$ and whose star graph is the incidence graph of a finite projective plane. Edjvet and Vdovina generalized this to (m, k) -special presentations and we generalize it to (m, k, ν) -special presentations. These are group presentations in which each relator has length k and whose star graph has ν isomorphic components, each of which is the incidence graph of a generalized m -gon. Groups defined by (m, k, ν) -special cyclic presentations act on Euclidean or hyperbolic buildings.

A cyclic presentation of a group is a group presentation with an equal number of generators and relators that admits a cyclic symmetry. In this talk I will discuss work with Ihechukwu Chinyere in which we classify the (m, k, ν) -special cyclic presentations and investigate SQ-universality of the groups they define.